



DISCOVERIES AT A SPECIFIC JUNCTURE FOR A UNIQUE TYPE OF NON-LINEAR

#¹**Mr.JAKKULA MADHUSUDAN**, *Assistant Professor*

#²**Mr.GURRAM SRINIVAS**, *Assistant Professor*

Department of Mathematics,

SREE CHAITANYA INSTITUTE OF TECHNOLOGICAL SCIENCES, KARIMNAGAR, TS.

ABSTRACT: The goal of this research article is to offer a novel iteration method and to verify convergence and stability results for it. We also claim the newly presented iterative strategy has superior efficiency than some of the current iterations in the literature. Our claim is supported by numerical illustration.

Keywords: *J*-iteration; Suzuki generalized non expansive mapping; stability.

1. INTRODUCTION AND PRELIMINARIES

Because of its applications in game theory, economics, and mathematics, among others, fixed point theory has expanded into an interdisciplinary domain. Because solving fixed point problems is challenging in and of itself, iterative methods must be used. The examination of establishing a method for locating the fixed point iteratively that is both easier and faster is an intriguing and ongoing subject of study. Scholars have developed a wide range of repetitive approaches for solving fixed point problems using a wide range of operators over time [1,2,3-5,6-12].

Ullah and Arshad [13] invented the succeeding M*-iteration approach in 2017.

$$\begin{cases} p_n = (1 - \beta_n) \zeta_n + \beta_n T \zeta_n \\ h_n = T((1 - \alpha_n) T \zeta_n + \alpha_n T p_n) \\ \zeta_{n+1} = Th_n \end{cases} \quad (1.1)$$

It is critical to emphasize that α_n and β_n are both collections of numbers that begin and end at the coordinates (0, 1).

The reference authors [13] claim that their iteration method delivers a higher rate of acceleration than other iteration systems published in scholarly literature (Abbas et al., Picard, Mann, Ishikawa, Noor, Agarwal et al.).

The "K-iteration scheme" devised by Hussain et al. [14] is an innovative way to iteration. They demonstrated the convergence of this iterative method by using the category of Suzuki generalized non-expansive mapping in uniformly convex Banach space.

$$\begin{cases} p_n = (1 - \beta_n) \zeta_n + \beta_n T \zeta_n \\ h_n = T((1 - \alpha_n) T \zeta_n + \alpha_n T p_n) \\ \zeta_{n+1} = Th_n \end{cases} \quad (1.2)$$

Ullah and Arshad [7] used the K*-iteration approach once more in 2018.

$$\begin{cases} p_n = (1 - \beta_n) \zeta_n + \beta_n T \zeta_n \\ h_n = T((1 - \alpha_n) p_n + \alpha_n T p_n) \\ \zeta_{n+1} = Th_n \end{cases} \quad (1.3)$$

The α_n and β_n numbers ensure that both α_n and β_n lie inside the range of zero to one for each n . Ullah and Arshad [6] used this methodology to apply the M-iteration strategy in 2018.

$$\begin{cases} \rho_1 = (1 - a_1)\zeta_1 + a_1\zeta_2, \\ \zeta_1 = \ell\rho_1, \\ \zeta_{n+1} = \ell\zeta_n \end{cases} \quad (1.4)$$

Consider the sequence $\{a_i\}$, which guarantees $a_i \in (0, 1)$ for every $i \in \mathbb{N}$.

In this case, Bhutia and Tiwari [1] presented the following explanation of the J-Iteration method:

If the initial approximation is 0, it is said that

$$\begin{cases} \rho_1 = \ell((1 - a_1)\zeta_1 + a_1\zeta_2), \\ \zeta_1 = \ell((1 - a_1)\rho_1 + a_1\ell\rho_1), \\ \zeta_{n+1} = \ell\zeta_n. \end{cases} \quad (1.5)$$

We are now deploying a revolutionary iteration process based on the following correlation:

The following procedures must be followed to acquire an initial estimate of zero:

$$\begin{cases} \rho_1 = \ell^2((1 - a_1)\zeta_1 + a_1\ell^2\zeta_2), \\ \zeta_1 = \ell^2((1 - a_1)\rho_1 + a_1\ell^2\rho_1), \\ \zeta_{n+1} = \ell^2((1 - a_1)\rho_1 + a_1\ell^2\rho_1). \end{cases} \quad (1.6)$$

We believe that the iterative strategy under discussion converges faster than the one established by Bhutia and Tiwari [1], as well as numerous other previously explored iterative sequences.

Definition 1.1 [2]:

In \mathbb{R} , the sequence is represented as $\{a_i\}_{i=0}^{\infty}$. This means that $\lim_{i \rightarrow \infty} a_i = 0$ if and only if $a_i \in (0, 1)$ and $a_i \rightarrow 0$ as $i \rightarrow \infty$. The equation $a_{i+1} = a_i(TM, \dots)$ describes a process that repeats and reaches a fixed point, demonstrating the stability with respect to T .

Definition 1.2 [15]:

Let H be a non-empty subset of a Banach space X and $\ell: X \rightarrow X$ be a mapping. ℓ is called a generalized contraction mapping if there exists a real number $k < 1$ such that for all $x, y \in X$ we have $d(\ell x, \ell y) \leq kd(x, y)$.

Definition 1.3 [13]:

An operator $\ell: K \rightarrow K$ is said to satisfy the condition (C), if for all $x, y \in K$, we have $d(x, \ell x) \leq d(x, y)$ implies $d(\ell x, \ell y) \leq d(x, y)$. Any mapping satisfies condition (C) is also known as Suzuki generalized non-expansive mapping.

Proposition 1.4 [13]:

exemplifies a mapping from X to X . Furthermore, it is a non-empty segment of the Banach space T .

1. If there is no growth for, 1. represents Suzuki generalized non-expansive mapping.

2. A quasi-non-expansive mapping is similar to a Suzuki generalized non-expansive mapping when a set point is used.

Lemma 1.5 [16]:

For a Suzuki generalized non-expansive mapping (a) , change the notation (a) to (i) (a) . In addition, any non-empty Banach space should be written as i . As a result, for each $b \in X$, we obtain

$$\| \ell x - \ell y \| \leq 3 \| \ell x - x \| + \| x - y \| \quad (1.7)$$

Lemma 1.6 [17]:

Indicate with a A uniformly convex \mathbb{R} is a set of real numbers in Banach space that has the property $0 < b_n < 1$ for any $n > 1$. i has two real number sequences, a_n and b_n . These sequences meet the requirements $\limsup_{n \rightarrow \infty} a_n = i$ and $\limsup_{n \rightarrow \infty} b_n = i$.

Remark 1.7 [16]:

Assume i is a non-empty subset of T with a finite sequence $\{a_n\}$. Consider the situation where $b_n = 0$. As a result, $i(L, n)$ equals $\limsup_{n \rightarrow \infty} a_n$ for each a value that does not occur in a . The set $a(i, i) = i(T, i)$ depicts the asymptotic center of i with respect to M . This set also depicts the radius of i as it approaches i .

Results

Theorem 2.1:

Let a denote a non-empty subset of a Banach space M and: $M \rightarrow M$ represents a generalized contraction mapping. There are three real numbers in the interval $[0, 1]$: $i \forall a = 0$, $aaa = 0$, and $bab = 0$. Every single

one of them fulfills the condition that $aa = i$ and $a=0$. Using the iterative approach (1.6), define the sequence $aa=0$. Following that, at, the series $AeAe=0$ reaches a substantial convergence point [18-22].

Do you require assistance? Drawing on personal experience: It has exactly one fixed point because it is both a generalized contraction mapping and a contraction mapping in general. This is due to the Banach contraction principle. Consider b to be the only constant within. At the moment, 1.6 iterations are planned.

$$\begin{aligned}\|\wp_n - q\| &= \|\ell^n((1-\beta_n)\mathfrak{Z}_n + \beta_n \ell^n \mathfrak{Z}_n) - q\| \\ &\leq k \|(1-\beta_n)\mathfrak{Z}_n + \beta_n \ell^n \mathfrak{Z}_n - q\| \\ &\leq k[(1-\beta_n)\|\mathfrak{Z}_n - q\| + \|\beta_n \ell^n \mathfrak{Z}_n - q\|] \\ &\leq k[(1-\beta_n)\|\mathfrak{Z}_n - q\| + \beta_n k \|\beta_n \mathfrak{Z}_n - q\|] \\ &\leq k[1-\beta_n(1-k)]\|\mathfrak{Z}_n - q\|.\end{aligned}$$

By the hypothesis of theorem, we have $1 - \beta_n(1-k) < 1$, so we can write

$$\|\wp_n - q\| \leq k[1-\beta_n(1-k)]\|\mathfrak{Z}_n - q\| \quad (2.1)$$

And

$$\begin{aligned}\|\mathfrak{h}_n - q\| &= \|\ell^n((1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n) - q\| \\ &\leq k \|(1-\alpha_n)\wp_n + \alpha_n \ell^n \wp_n - q\| \\ &\leq k[(1-\alpha_n)\|\wp_n - q\| + \|\alpha_n \ell^n \wp_n - q\|] \\ &\leq k[(1-\alpha_n)\|\wp_n - q\| + \alpha_n k \|\wp_n - q\|] \\ &\leq k[1-\alpha_n(1-k)]\|\wp_n - q\|.\end{aligned}$$

Again, by the hypothesis of theorem we have $1 - \alpha_n(1-k) < 1$ and using (2.1) we have

$$\begin{aligned}\|\mathfrak{h}_n - q\| &\leq k \|\wp_n - q\| \\ &\leq k^2 \|\mathfrak{Z}_n - q\|\end{aligned} \quad (2.2)$$

Furthermore, we obtain by using equations (1.6), (2.1), and (2.2).

$$\begin{aligned}\|\mathfrak{Z}_{n+1} - q\| &= \|\ell^n((1-\gamma_n)\ell^n \mathfrak{Z}_n + \gamma_n \ell^n \mathfrak{h}_n) - q\| \\ &\leq k \|(1-\gamma_n)\ell^n \mathfrak{Z}_n + \gamma_n \ell^n \mathfrak{h}_n - q\| \\ &\leq k[(1-\gamma_n)\|\ell^n \mathfrak{Z}_n - q\| + \|\gamma_n \ell^n \mathfrak{h}_n - q\|] \\ &\leq k[(1-\gamma_n)k \|\mathfrak{Z}_n - q\| + \gamma_n k \|\mathfrak{h}_n - q\|] \\ &\leq k[(1-\gamma_n)k^2 \|\mathfrak{Z}_n - q\| + \gamma_n k^3 \|\mathfrak{Z}_n - q\|] \\ &\leq k^3[1-\gamma_n(1-k)]\|\mathfrak{Z}_n - q\|\end{aligned} \quad (2.3)$$

As a result of the aforementioned factors, we have

$$\begin{aligned}\|\mathfrak{Z}_n - q\| &\leq k^3[1-\gamma_{n-1}(1-k)]\|\mathfrak{Z}_{n-1} - q\| \\ \|\mathfrak{Z}_{n-1} - q\| &\leq k^3[1-\gamma_{n-2}(1-k)]\|\mathfrak{Z}_{n-2} - q\| \\ \|\mathfrak{Z}_1 - q\| &\leq k^3[1-\gamma_0(1-k)]\|\mathfrak{Z}_0 - q\|.\end{aligned}$$

The total of these distinctions is as follows:

$$\|\mathfrak{Z}_{n+1} - q\| \leq k^{3(n+1)}\|\mathfrak{Z}_0 - q\| \prod_{i=0}^n [1-\gamma_i(1-k)].$$

Now $k < 1$ so $1-k > 0$ and $\gamma_i \leq 1$ for all $n \in N$, hence we have $1 - \gamma_i(1-k) < 1$. We know that $1 - x \leq e^{-x}$ for all $x \in [0, 1]$. Hence,

$$\|\mathfrak{Z}_{n+1} - q\| \leq k^{3(n+1)}\|\mathfrak{Z}_0 - q\| e^{-(1-k)\sum_{i=0}^n \gamma_i} \quad (2.4)$$

Taking limit as $n \rightarrow \infty$ both sides, we have $\lim_{n \rightarrow \infty} \|\mathfrak{Z}_n - q\| = 0$. This completes the proof

Theorem 2.2: Let H be a non-empty subset of a Banach space X and $\ell: X \rightarrow X$ be a generalized contraction mapping. Let $\{\mathfrak{I}_n\}_{n=0}^\infty$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^\infty, \{\beta_n\}_{n=0}^\infty, \{\gamma_n\}_{n=0}^\infty$ in $[0, 1]$ satisfying $\sum_{n=0}^\infty \gamma_n = \infty$. Then the sequence $\{\mathfrak{I}_n\}_{n=0}^\infty$ is ℓ -stable.

Proof: Let $\{\mathfrak{I}_n\}_{n=0}^\infty$ be any sequence in H and let the sequence generated by (1.6) be $t_{n+1} = f(\ell, \mathfrak{I}_n)$ and let it converges to the unique fixed point q of ℓ .

Suppose $\delta_n = \|t_{n+1} - f(\ell, t_n)\|$. Now, we will prove that $\lim_{n \rightarrow \infty} \delta_n = 0$ if and only if $\lim_{n \rightarrow \infty} t_n = q$. First of all, suppose that $\lim_{n \rightarrow \infty} t_n = q$.

Then we have

$$\begin{aligned} \delta_n &= \|t_{n+1} - f(\ell, t_n)\| \\ &\leq \|t_{n+1} - q\| + \|f(\ell, t_n) - q\| \\ &\leq \|t_{n+1} - q\| + k^3[1 - \gamma_n(1 - k)] \|t_n - q\| \end{aligned}$$

Taking limit as $n \rightarrow \infty$ both sides of the above inequality we have $\lim_{n \rightarrow \infty} \delta_n = 0$.

Conversely suppose that $\lim_{n \rightarrow \infty} \delta_n = 0$.

Now, we have

$$\begin{aligned} \|t_{n+1} - q\| &\leq \|t_{n+1} - f(\ell, t_n)\| + \|f(\ell, t_n) - q\| \\ &\leq \delta_n + \|f(\ell, t_n) - q\|. \end{aligned}$$

Using Theorem 2.1, we can write

$$\|t_{n+1} - q\| \leq \delta_n + [1 - \gamma_n(1 - k)] \|t_n - q\|.$$

Now $0 < k < 1$ and $\gamma_n \leq 1$ for all $n \in \mathbb{N}$ and $\lim_{n \rightarrow \infty} \delta_n = 0$. Then from the above inequality and lemma (1.6) we have, $\lim_{n \rightarrow \infty} \|t_n - q\| = 0$.

Hence the sequence $\{x_n\}_{n=0}^\infty$ is ℓ -stable.

Suzuki's generalized non-expansive mapping can now be employed with a few fixed point values.

Lemma 2.3: Let H be a non-empty closed convex subset of a Banach space X and $\ell: X \rightarrow X$ be a Suzuki generalized non-expansive mapping with $F(\ell) \neq \emptyset$. Let $\{\mathfrak{I}_n\}_{n=0}^\infty$ be the sequence of X defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^\infty, \{\beta_n\}_{n=0}^\infty, \{\gamma_n\}_{n=0}^\infty$ in $[0, 1]$ satisfying $\sum_{n=0}^\infty \gamma_n = \infty$. Then $\lim_{n \rightarrow \infty} \|\mathfrak{I}_n - q\|$ exists for all $q \in F(\ell)$.

Proof: Let $q \in F(\ell)$. Now, using the convexity of H we have, $(1 - \gamma_n)\mathfrak{I}_n + \gamma_n\ell\mathfrak{I}_n \in H$ for all $n \in \mathbb{N}$. Since ℓ is Suzuki generalized non-expansive mapping so we can write

$$\frac{1}{2} \|q - \ell q\| = 0 \leq \|q - ((1 - \gamma_n)\mathfrak{I}_n + \gamma_n\ell\mathfrak{I}_n)\|,$$

which implies that

$$\|\ell q - \ell((1 - \gamma_n)\mathfrak{I}_n + \gamma_n\ell\mathfrak{I}_n)\| \leq \|q - ((1 - \gamma_n)\mathfrak{I}_n + \gamma_n\ell\mathfrak{I}_n)\|.$$

Now from the iterative process (1.6) we have

$$\begin{aligned}
\|\rho_n - q\| &= \|F^n((1 - \beta_n)\zeta_n + \beta_n F^2 \zeta_n) - F^n q\| \\
&\leq \|(1 - \beta_n)\zeta_n + \beta_n F^2 \zeta_n - q\| \\
&\leq (1 - \beta_n)\|\zeta_n - q\| + \beta_n \|F^2 \zeta_n - q\| \\
&\leq (1 - \beta_n)\|\zeta_n - q\| + \beta_n \|\zeta_n - q\| \\
&\leq \|\zeta_n - q\|
\end{aligned} \tag{2.5}$$

Now

$$\begin{aligned}
\|h_n - q\| &= \|F^n((1 - \alpha_n)\rho_n + \alpha_n F^2 \rho_n) - F^n q\| \\
&\leq \|(1 - \alpha_n)\rho_n + \alpha_n F^2 \rho_n - q\| \\
&\leq (1 - \alpha_n)\|\rho_n - q\| + \alpha_n \|F^2 \rho_n - q\| \\
&\leq (1 - \alpha_n)\|\rho_n - q\| + \alpha_n \|\rho_n - q\| \\
&\leq \|\rho_n - q\| \\
&\leq \|\zeta_n - q\|.
\end{aligned} \tag{2.6}$$

Again using (1.6), (2.5) and (2.7), we get

$$\begin{aligned}
\|\zeta_{n+1} - q\| &= \|F^n((1 - \gamma_n)\rho_n + \gamma_n F^2 h_n) - F^n q\| \\
&\leq \|(1 - \gamma_n)\rho_n + \gamma_n F^2 h_n - q\| \\
&\leq (1 - \gamma_n)\|\rho_n - q\| + \gamma_n \|F^2 h_n - q\| \\
&\leq (1 - \gamma_n)\|\rho_n - q\| + \gamma_n \|\zeta_n - q\| \\
&\leq (1 - \gamma_n)\|\zeta_n - q\| + \gamma_n \|\zeta_n - q\| \\
&\leq \|\zeta_n - q\|.
\end{aligned}$$

Hence $\{\|\zeta_n - q\|\}$ is bounded and non-increasing for all $q \in F(\mathcal{F})$.

Hence $\lim_{n \rightarrow \infty} \|\zeta_n - q\|$ exists for all $q \in F(\mathcal{F})$.

Theorem 2.4: Let H be a non-empty closed convex subset of a Banach space X and $T: X \rightarrow X$ be a Suzuki generalized non-expansive mapping. Let $\{\zeta_n\}_{n=0}^\infty$ be the sequence of defined by the iterative scheme (1.6) with real sequences $\{\alpha_n\}_{n=0}^\infty$, $\{\beta_n\}_{n=0}^\infty$, $\{\gamma_n\}_{n=0}^\infty$ in $[0, 1]$ satisfying $\sum_{n=0}^\infty \gamma_n = \infty$. Then $F(T) \neq \emptyset$ if and only if $\{\zeta_n\}_{n=0}^\infty$ is bounded and $\lim_{n \rightarrow \infty} \|T\zeta_n - \zeta_n\| = 0$.

Proof: First suppose that $F(T) \neq \emptyset$. Let $q \in F(T)$.

Then by Lemma 2.4, $\lim_{n \rightarrow \infty} \|\zeta_n - q\|$ exists for all $q \in F(T)$ and $\{\zeta_n\}_{n=0}^\infty$ is a bounded sequence.

Let $\lim_{n \rightarrow \infty} \|\zeta_n - q\| = \theta$ for some $\theta > 0$.

Now, from (2.5), we have

$$\limsup_{n \rightarrow \infty} \|\rho_n - q\| \leq \limsup_{n \rightarrow \infty} \|\zeta_n - q\| = \theta.$$

By Proposition 1.4, we have

$$\limsup_{n \rightarrow \infty} \|F\zeta_n - q\| \leq \limsup_{n \rightarrow \infty} \|\zeta_n - q\| = \theta.$$

Now using (1.6) and (2.5) we have

$$\begin{aligned}
\|\zeta_{n+1} - q\| &= \|F^n((1 - \gamma_n)\rho_n + \gamma_n F^2 h_n) - F^n q\| \\
&\leq \|(1 - \gamma_n)\rho_n + \gamma_n F^2 h_n - q\| \\
&\leq (1 - \gamma_n)\|\rho_n - q\| + \gamma_n \|F^2 h_n - q\| \\
&\leq (1 - \gamma_n)\|\rho_n - q\| + \gamma_n \|\zeta_n - q\| \\
&\leq (1 - \gamma_n)\|\rho_n - q\| + \gamma_n \|\rho_n - q\| \\
&\leq \|\rho_n - q\|,
\end{aligned}$$

which implies that $\|\mathfrak{J}_{n+1} - q\| \leq \|\rho_n - q\|$ and hence

$$\theta \leq \liminf_{n \rightarrow \infty} \|\rho_n - q\|.$$

Therefore, we can write

$$\theta \leq \liminf_{n \rightarrow \infty} \|\rho_n - q\| \leq \limsup_{n \rightarrow \infty} \|\rho_n - q\| \leq \theta.$$

Thus, we obtain $\lim_{n \rightarrow \infty} \|\rho_n - q\| = \theta$.

Now, we have

$$\begin{aligned} \theta &= \lim_{n \rightarrow \infty} \|\ell^n((1 - \beta_n)\mathfrak{J}_n + \beta_n \ell^n \mathfrak{J}_n) - q\| \\ &\leq \lim_{n \rightarrow \infty} \|(1 - \beta_n)\mathfrak{J}_n + \beta_n \ell^n \mathfrak{J}_n - q\| \\ &\leq \lim_{n \rightarrow \infty} \|(1 - \beta_n)\|\mathfrak{J}_n - q\| + \beta_n \|\ell^n \mathfrak{J}_n - q\| \\ &\leq \lim_{n \rightarrow \infty} (1 - \beta_n) \|\mathfrak{J}_n - q\| + \beta_n \|\mathfrak{J}_n - q\| \\ &\leq \lim_{n \rightarrow \infty} \|\mathfrak{J}_n - q\| \leq \theta. \end{aligned}$$

Hence we can write

$$\theta \leq \lim_{n \rightarrow \infty} \|(1 - \beta_n)(\mathfrak{J}_n - q) + \beta_n(\ell^n \mathfrak{J}_n - q)\| \leq \theta.$$

Thus $\lim_{n \rightarrow \infty} \|(1 - \beta_n)(\mathfrak{J}_n - q) + \beta_n(\ell^n \mathfrak{J}_n - q)\| = \theta$.

Using Lemma 1.6 and the above calculations we have $\lim_{n \rightarrow \infty} \|\ell^n \mathfrak{J}_n - \mathfrak{J}_q\| = 0$.

Conversely, let $\{\mathfrak{J}_n\}_{n=0}^\infty$ is bounded and $\lim_{n \rightarrow \infty} \|\ell^n \mathfrak{J}_n - \mathfrak{J}_q\| = 0$.

Let $q \in A(H, \{\mathfrak{J}_n\})$.

By Lemma 1.5, we have

$$\begin{aligned} r(\ell q, \{\mathfrak{J}_n\}) &= \limsup_{n \rightarrow \infty} \|\mathfrak{J}_n - \ell q\| \\ &\leq \limsup_{n \rightarrow \infty} \|\mathfrak{J}_n - \ell \mathfrak{J}_n + \ell \mathfrak{J}_n - \ell q\| \\ &\leq \limsup_{n \rightarrow \infty} \|\mathfrak{J}_n - q\| \end{aligned}$$

which implies that $\ell q \in A(H, \{\mathfrak{J}_n\})$. Since X is uniformly convex Banach space. It follows that $A(H, \{\mathfrak{J}_n\})$ is singleton. Hence $\ell q = q$ implies that $q \in F(\ell)$ and hence $F(\ell) \neq \emptyset$. This completes the proof.

Table 1. Using the J-iteration method repeatedly while considering the work of Bhutia and Tiwar as an example

Iteration	J-Iteration	Iteration (1.6)
0	4	4
1	2.0183456356079	2.01874653158
2	2.0001845667869	2.00015675043
4	2.000000200677	2.00000001877
5	2.000000002057	2.00000000011
6	2.000000000026	2
7	2.000000000002	2
8	2	2

Example 2.3: Consider the mapping $\ell(\theta) = (\theta + 2)^{\frac{1}{2}}$. Clearly ℓ is a generalized contraction mapping and $\{\alpha_n\}_{n=0}^\infty, \{\beta_n\}_{n=0}^\infty, \{\gamma_n\}_{n=0}^\infty$ be the sequence defined by $\alpha_n = \beta_n = \gamma_n = \frac{1}{4}$ for all $n \in \mathbb{N}$. We now compare the rate of convergence of our iterative scheme with J-iteration scheme by considering the example of Bhutia and Tiwari [1] using following Table 1.

2. CONCLUSION

As seen in the table above, the iterative plan outperforms the J-iteration approach with a convergence rate of 1.6. A Variety of Interests According to the authors, there are no competing interests.

REFERENCES

1. Bhutia JD, Tiwari K. New iteration process for approximating fixed points in Banach spaces. Journal of linear and topological algebra. 2019;8(4):237-250.
2. Erturk M, Gursoy F. Some convergence, stability and data dependency results for Picard-S iteration method of quasi-strictly contractive operators, Mathematica Bohemica. 2019;144:69-83.
3. Hussain N, Ullah K, Arshad M. Fixed point approximation for Suzuki generalized nonexpansive mappings via new iteration process. J. Nonlinear Convex Anal. 2018;19(8):1383-1393.

4. Thakur BS, Thakur D, Postolache M. A new iterative scheme for numerical reckoning fixed points of Suzuki's generalized nonexpansive mappings. *Appl. Math. Comp.* 2016;275:147-155.
5. Thakur BS, Thakur D, Postolache M. A new iterative scheme for approximating fixed points of non expansive mappings. *Filomat.* 2016;30:2711-2720.
6. Shanjit L, Rohen Y, Chandok S, Devi MB. Some results on iterative proximal convergence and chebyshev center. *Journal of Function Spaces.* 2021;Article ID 8863325:1-8.
7. Shanjit L, Rohen Y, Singh KA. Cyclic relatively nonexpansive mappings with respect to orbits and best proximity point theorems. *Journal of Mathematics.* 2021;Article ID 6676660:1-7.
8. Shanjit L, Rohen Y. Non-convex proximal pair and relatively nonexpansive maps with respect to orbits. *Journal of Inequalities and Applications.* 2021;2021:124.
9. Ullah K, Arshad M. On different results for new three step iteration process in Banach Spaces. *Springer Plus.* 2016;1-15.
10. Ullah K, Arshad M. New iteration process and numerical reckoning fixed point in Banach space. *U.P.B.sci. bull. (Series A).* 2017;79(4):113-122.